Optimal control applied to Anthrax transmission in animal populations

Ana-Maria Croicu
Kennesaw State University

Anthrax is an infectious disease that is known to infect both humans and animals. It turns out that Anthrax outbreaks occur periodically animals. A vaccine against the Anthrax disease was developed long time ago. The purpose of this research is to identify an optimal vaccination regime based on the classical optimal control theory. For this goal, the Anthrax disease is considered for two special cases: herbivore and carnivore populations.

Bilinear operators arising from geometric measure theory

Alex Iosevich
University of Rochester

We are going describe the sharp Lebesgue space bounds for the bilinear operator

$$B(f,g)(x) = \int f(x-y)g(x-\theta y)d\sigma(y),$$

where $\theta \in O_2(\mathbb{R})$ and σ is the measure on the unit circle S^1. This operator arises in the study of congruence classes of triangles in geometric measure theory and geometric combinatorics. Related questions will be examined and connections established with the Fourier Integral Operator theory in the linear setting.

Comparing the degrees of unconstrained and constrained approximation by polynomials

D. Leviatan
Tel Aviv University, Israel

It is quite obvious that one should expect that the degree of constrained approximation be worse than the degree of unconstrained approximation. However, it turns out that in certain cases we can deduce the behavior of the degrees of the former from information about the latter.
Let $E_n(f)$ denote the degree of approximation of $f \in C[-1,1]$, by algebraic polynomials of degree $< n$, and assume that we know that for some $\alpha > 0$ and $N \geq 1$,

$$n^\alpha E_n(f) \leq 1, \quad n \geq N.$$

Suppose that $f \in C[-1,1]$, changes its monotonicity or convexity $s \geq 0$ times in $[-1,1]$ ($s = 0$ means that f is monotone or convex, respectively). We are interested in what may be said about its degree of approximation by polynomials of degree $< n$ that are comonotone or coconvex with f. Specifically, if f changes its monotonicity or convexity at $Y_s := \{y_1, \dots, y_s\}$ ($Y_0 = \emptyset$) and the degrees of comonotone and coconvex approximation are denoted by $E_n^{(q)}(f, Y_s)$, $q = 1, 2$, respectively. We investigate when can one say that

$$n^\alpha E_n^{(q)}(f, Y_s) \leq c(\alpha, s, N), \quad n \geq N^*,$$

for some N^*. Clearly, N^*, if it exists at all (we prove it always does), depends on α, s and N. However, it turns out that for certain values of α, s and N, N^* depends also on Y_s, and in some cases even on f itself, and this dependence is essential.

Weighted polynomial approximation

Kirill Kopotun
University of Manitoba, Canada

In this talk, we'll discuss matching direct and inverse results for weighted approximation by algebraic polynomials in the L_p, $0 < p \le \infty$, (quasi)norm weighted by

- certain "averages" of doubling weights, or
- doubling weights having finitely many zeros and singularities and not "too rapidly changing" away from those points.

In particular, we'll discuss why "averages" of A^* weights behave like constant weights in the corresponding inverse estimates, and why the "averages" of general doubling weights do not. Among other things, we close a gap left in a 2001 paper by G. Mastroianni and V. Totik.

Various equivalence type results involving related K-functionals and realization type results (obtained as corollaries of our estimates) will also be discussed.

Inequalities of Jackson–Stechkin type for approximation of elements of Hilbert space

Vladislav Babenko
DNU, Ukraine

We introduce a new characteristic of an element in a Hilbert space that is a generalized moduli of continuity $\omega_\varphi(x; L_{p,V}([0,\delta]))$ and we obtain new exact inequalities of Jackson–Stechkin type with these moduli of continuity for the approximation of elements of a Hilbert space. These results include many well-known inequalities for approximation of periodic functions by trigonometric polynomials, approximation of non-periodic functions by entire functions of exponential type, similar results for almost periodic functions, and others. A number of results is new even in these classical situations.

Linear independence of time-frequency translates

Chris Heil
Georgia Institute of Technology

The Linear Independence of Time-Frequency Translates Conjecture, also known as the HRT conjecture, states that any finite set of time-frequency translates of a given square-integrable function must be linearly independent. This conjecture, first stated in print in 1996, remains open today. We will discuss this conjecture, its context, and the (frustratingly few) partial results that are currently available.

On conservation laws of Navier-Stokes Galerkin discretizations

Leo Rebholz
Clemson University

We study conservation properties of Galerkin methods for the incompressible Navier-Stokes equations, without the divergence constraint strongly enforced. In typical discretizations such as the mixed finite element method, the conservation of mass is enforced only weakly, and this leads to discrete solutions which may not conserve energy, momentum, angular momentum, helicity, or vorticity, even though the physics of the Navier-Stokes equations dictate that they should. We aim in this work to construct discrete formulations that conserve as many physical laws as possible without utilizing a strong enforcement of the divergence constraint, and doing so leads us to a new formulation that conserves each of energy, momentum, angular momentum, enstrophy in 2D, helicity and vorticity (for reference, the usual convective formulation does not conserve most of these quantities). Several numerical experiments are performed, which verify the theory and test the new formulation.

Computational hemodynamics for Computer Aided Clinical Trials: looking at the theory, struggling with the practice

Alessandro Veneziani
Emory University – IUSS Pavia

When we get to the point of including the huge and relevant experience of finite element fluid modeling collected in over 25 years of experience in the treatment of cardiovascular diseases, the risk of getting "lost in translation" is real. The most important issues are the *reliability* that we need to guarantee to provide a trustworthy decision support to clinicians; the *efficiency* we need to guarantee to fit into the demand coming from a large volume of patients in Computer Aided Clinical Trials as well as short timelines required by special circumstances (emergency) in Surgical Planning.

In this talk, we will report on some recent activities taken at Emory to make this transition possible. Reliability requirements call for an appropriate integration of measurements and numerical models, as well as for uncertainty quantification. In particular, image and data processing are critical to feeding mathematical models. However, there are several challenges still open, e.g. in simulating blood flow in patient-specific arteries after stent deployment; or in assessing the correct boundary data set to be prescribed in complex vascular districts. The gap between theory, in this case, is apparent and *good simulation and assimilation practices* in finite elements for clinical hemodynamics need to be drawn. The talk will cover these topics.

For computational efficiency, we will cover some numerical techniques currently in use for coronary blood flow, like the *Hierarchical Model Reduction* or efficient methods for coping with turbulence in aortic flows.

As Clinical Trials are currently one of the most important sources of information for medical research and practice, we envision that the suitable achievement of reliability and efficiency requirements will make *Computer Aided* Clinical Trials (specifically with a strong Finite-Elements-in-Fluids component) an important source of information with a significant impact on the quality of healthcare.

This is a joint work with the scholars and students of the Emory Center for Mathematics and Computing in Medicine ($E(CM)^2$), the Emory Biomech Core Lab (Don Giddens and Habib Samady), the Beta-Lab at the University of Pavia (F. Auricchio). This work is supported by the US National Science Foundation, Projects DMS 1419060, 1412963 1620406, Fondazione Cariplo, Abbott Vascular Inc., and the XSEDE Consortium.
